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Embedded Curved Boundaries and Adaptive Mesh Refinement

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The Embedded Curved Boundary method (ECB) is a new technique for treating complex boundaries on orthogonal simulation grids. ECB models curved boundaries with piecewise linear segments embedded in the grid. Generalization of the method to solutions across interfaces is described in Hewett & Kueny (these proceedings). Here we present quantitative comparisons of ECB to the traditional stairstep model in solving elliptic problems, and describe recent work on coupling ECB to Adaptive Mesh Refinement (AMR) methods. This is being done as part of an effort to incorporate ECB into existing simulation codes for a variety of applications.

A central virtue of ECB is that it allows modelling of complex boundaries on orthogonal, relatively coarse grids. Unstructured meshes may be used for the same purpose, but are harder to set up and not as easily parallelizable. Local refinement of rectangular grids can extend the capabilities of simple stairstep models, but this too adds complexity to the algorithms, and for many applications a stairstep model is wholly inadequate (for example, a stairstep model for a curved electrode surface will generate non-physical electric fields near the stairstep corners).

Figure 1 shows a comparison of both stairstep and ECB solutions with the analytic solution (A+B/R, where R is the distance from the origin) for the potential in a spherical cavity. Thd outer surface is held at 100 volts and the inner surface at 0 volts. Laplace's equation was solved on a 10 x 20 grid in

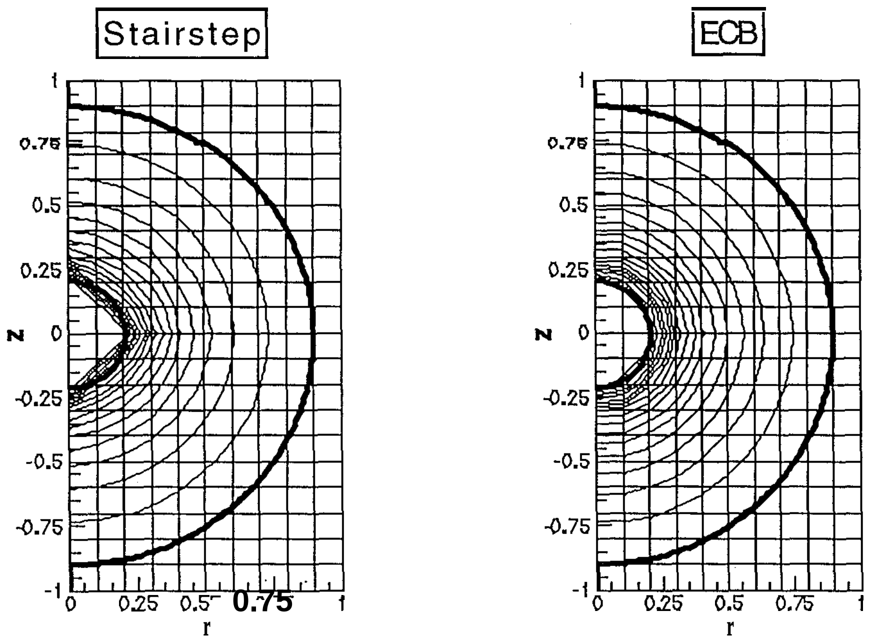

FIGURE 1

axisymmetric cylindrical coordinates using the Dynamic Alternating-Direction Implicit (DADI) method. (While DAD1 has so far been our method of choice, other elliptic solvers can be used.) The only significant ECB overhead occurs at the beginning of the simulation when information on the structures is accumulated and translated into the ECB representation; this information is then used to adjust coefficients of the finite-difference model used by the DAD1 solver. The ECB and stairstep models thus have comparable nmtimes, but ECB provides far more accurate results. If Figure 2 we plot contours of the difference between the computed solution and the analytic solution. For the stairstep model of boundaries, the maximum error was nearly 25% of the maximum potential, while ECB found the solution to better than 6% everywhere. In both cases the largest errors occurred in the vicinity of the small center electrode. ECB actually converged slightly faster than the stairstep method in this case (16 iterations vs. 18 for stairstep) which offset the extra time spent computing coeffkients. Increasing the number of grid points by a factor of four in each direction brought the stairstep code's error down to 8%, but using ECB on that grid reduced the error to nearly 1% (and did it in only 19 iterations vs. 33 for the stairstep code).

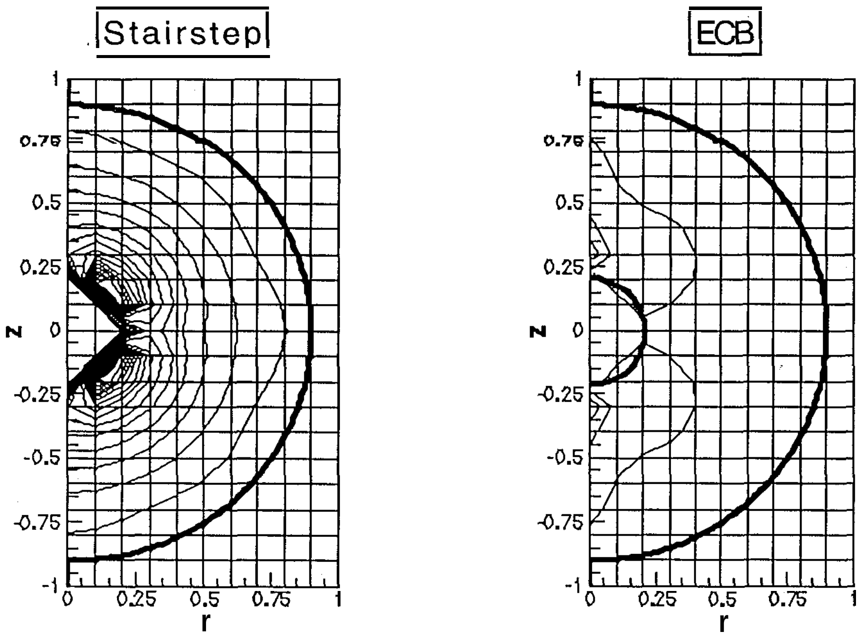

FIGURE 2

A common method of improving the accuracy of codes using stairstep models is local mesh refinement. In a region (such as near a structure surface) where finer resolution is required, an auxiliary, finer grid can be defmed. Solutions are obtained on both the coarse and fine grids, with appropriate communication of information at the boundaries.This provides resolution where needed, without wasting effort in regions without interesting details. We saw above that very fine grid spacing was needed in order for the stairstep boundary model to approach the accuracy of ECB in calculating potentials, so clearly ECB can decrease the level of mesh refinement that is needed in order to obtain a desired level of accuracy. At the same time, ECB itself can be used in conjunction with local mesh refinement to resolve details in specific regions. Figure 3 illustrates a case where a small structure that is not seen by a 20 x 20 grid is resolved by applying a small patch grid with twice the resolution. The data-passing algorithm between grids in this case was very simple:

1) A single AD1 sweep is done on the coarse grid.
2) Information from the coarse grid is copied onto corresponding fine grid points; the other fine grid points are filled in by interpolation.
3) A single AD1 sweep is done on the fine grid, keeping the values on its boundary fixed.
4) Those points just inside of the fine grid boundary which lie on top of coarse grid points have their information copied to the corresponding coarse grid points.
5) A single AD1 sweep is now done on the coarse grid, keeping those points just inside the fine grid boundary fixed. Coarse grid points lying *on* the fine grid boundary will now be affected by the Dirichlet boundary conditions just *inside* the fine grid boundary.
5) Repeat until solutions on both coarse and fine grids have converged to the desired level.

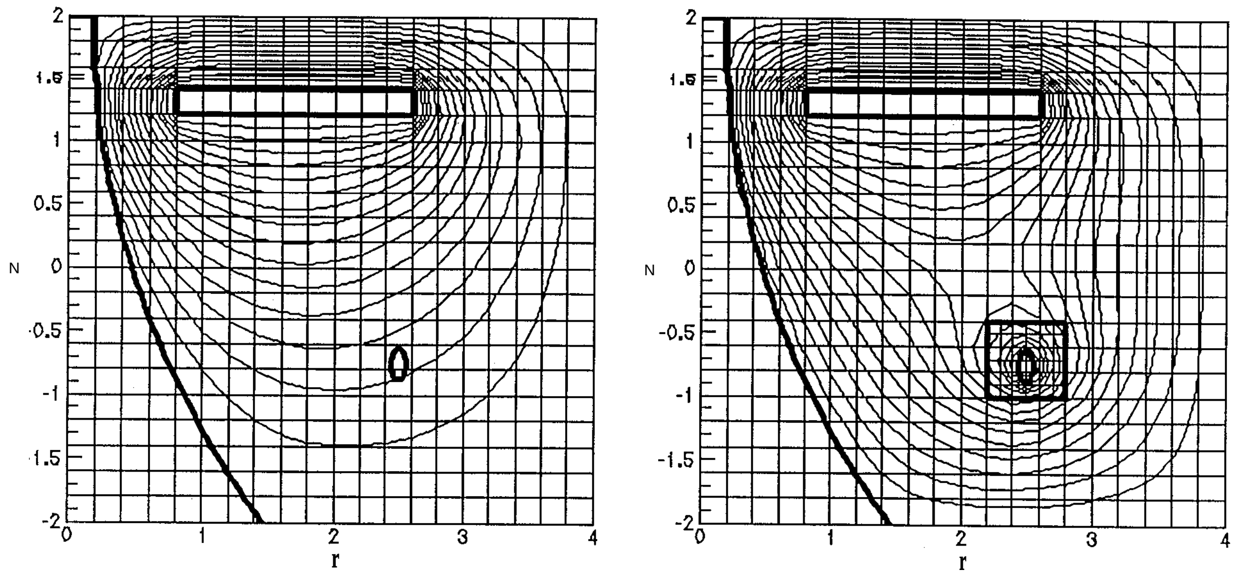

FIGURE 3

Convergence tends to be considerably faster than resolving the whole grid to the finest desired resolution, and further refinements of the method to increase efficiency are likely. More details may be resolved by adding successively finer grids one on top of another and passing information from the coarsest grid to the finest and back out again. The data-passing mechanism is very simple; an obvious refinement will be to add Neumann information at the fine grid boundary. Despite its simplicity, the method performs very well; the solution obtained using the patch grid in Figure 3 agrees to within 5% with that obtained on the corresponding points of a 200 x 200 grid. Comparisons of these results with those obtained using a stairstep model again showed that the stairstep model required several more levels of refinement in order to match the ECB results. We are currently determining how best to incorporate ECB into existing codes that already have well-developed AMR capabilities to build on.

We have demonstrated the advantages of the ECB method compared to the traditional stairstep model for representing structures on rectangular grids. ECB has provided better results in equal or less time on a number of model problems, while preserving the benefits of working on orthogonal grids. We have successfully coupled ECB to local mesh refinement methods, and we are currently working to incorporate it into existing codes that use AMR techniques. Other priorities in future research will include extending the ECB method from two dimensions to three, and adapting it to the treatment of moving boundaries.

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